# Antenna Structures: Evaluation of Reflector Surface Distortions

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*The reflector surface distortions of the 210-ft antenna as evaluated by the linearized formulation of the RMS paraboloid best-fitting computer program has provided sufficient significant digits in its answers for meaningful results. This article presents a clearer documentation as well as the error bounds of the formulation. Since basically the solution is a non-linear problem, improved formulation would be desirable. However, the program should be useful for evaluating larger than 210-ft antennas with about the same degree of distortion.*

## I. Introduction

Reflector surface distortions of antennas and their effects on the RF performance may be evaluated by best fitting to the distortions, in a least-squares pathlength sense, a paraboloid. The resulting value

$$\mathrm{rms} = \sqrt{\frac{\Sigma\,(\Delta PL_j)^2 A_i}{\Sigma A_i}}$$

is applicable in Ruze equation for computing the RF gain.

A computer program for this purpose was described earlier (Refs. 1 and 2). Results of its use with analytically computed distortions and with field measurements of the 210-ft antenna in calculating its RF performance have been reported (Refs. 3 and 4). Comparisons to RF performance measurements were made by Bathker (Ref. 5).

With the use of positional data of the best-fit paraboloid and the deflected positions of the RF feeds, the RF boresight directions may be calculated (Ref. 6).

To date, the best fitting of the analytical 210-ft antenna data using the linearized solution formulation has provided sufficient significant digits for meaningful results. Comparisons between analytical solutions and RF field tests have shown close correlations.

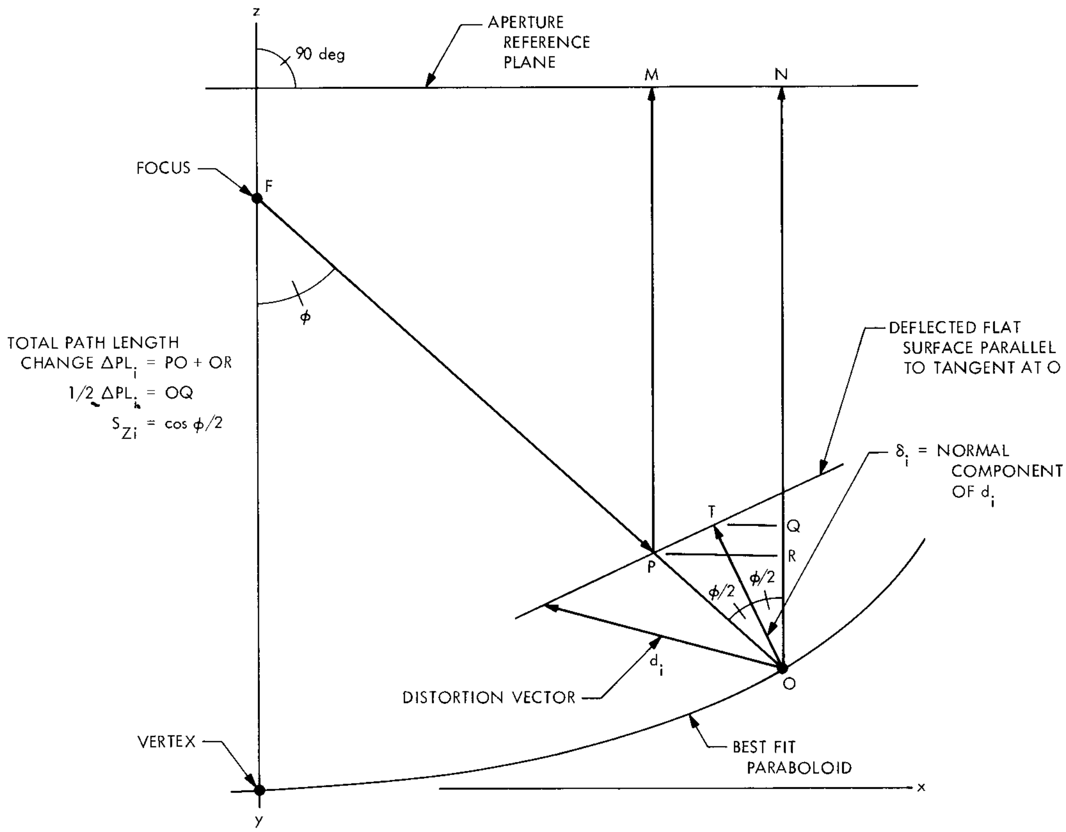


**Fig. 1. RF path-length change**

This article presents the results of studies made to determine the accuracy bounds of the best-fitting program and to present a clearer description of the formulation.

The documentation is presented as follows:

(1) The modifications, as well as the complete formulations, are graphically and algebraically delineated.

(2) Data manipulations to improve the accuracy of field measurements using the angle-measuring theodolite method will be described in a following article. The method combines the direction vectors from the analytical analysis with the direction lacking field measurements.

(3) The output has been converted to show SI (metric) units in addition to English units values.

## II. Formulation Modifications

From Refs. 1 and 2 the ½-path-length change or error may be denoted by

$$\Delta PL_i = -\delta_i \cdot \delta_{zi} \tag{1}$$

Fig. 1 shows the graphical definition of the equation. The basic assumption is made that the deflected surface is flat and has moved parallel to the tangent at the node O.

The error due to this assumption is illustrated in Fig. 2. A sample calculation using a point on the outer edge of the 210-ft dish results in a negligible error. For

$$\frac{\phi}{2} = 15 \text{ deg}$$

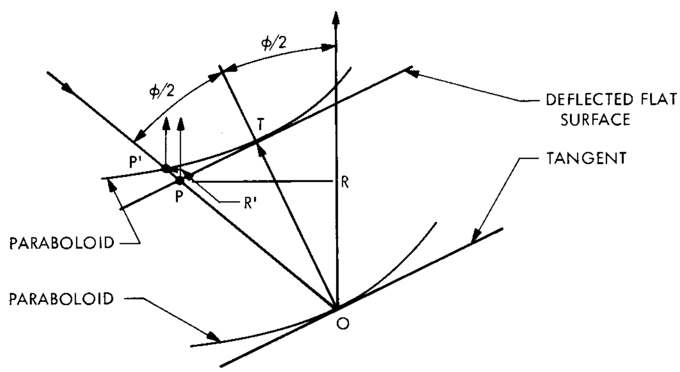


**Fig. 2. Assumption error**

and assuming a normal error (OT of Fig. 2) = 1.0 in., and an approximate radius of curvature = 2000 in., the path-length error $(P'P + PR') = 0.00004$ in.

The total normal distortion at a node on the surface of a reflector from the best-fit paraboloid is the sum of four types of normal errors:

$$S_i = S_{ia} + S_{ib} + S_{ic} + S_{id} \tag{2}$$

The first,

$$\begin{aligned} S_{ia} &= \text{surface distortions normal error} \\ &= n_i Sx_i + v_i Sy_i + w_i Sz_i \end{aligned} \tag{3}$$

The second,

$$\begin{aligned} S_{ib} &= \text{normal error due to change in focal length from the original paraboloid} \\ &= -K(x_i^2 + y_i^2)\, S_{zi} \end{aligned} \tag{4}$$

where

$$K = \frac{1}{4}\left(\frac{1}{F} - \frac{1}{F_n}\right)$$

$F$ = focal length of original paraboloid

$F_N$ = focal length of best fit paraboloid

Equation (4) is derived from the equation of the paraboloid

$$z_i = \frac{x_i^2 + y_i^2}{4F} \tag{5}$$

The change in $z$ from a change in $F$ results in

$$\Delta z_i = \frac{x_i^2 + y_i^2}{4}\left(\frac{1}{F} - \frac{1}{F_N}\right)$$

which is equivalent to

$$\Delta z_i = \frac{x_i^2 + y_i^2}{4F}\left(1 - \frac{F}{F_N}\right) \tag{6}$$

substituting Eqs. (5) into (6) and defining

$$K^1 = \left(1 - \frac{F}{F_N}\right)$$

yields

$$S_{ib} = -K^1 z_i s_{zi} \text{ (Fig. 3)} \tag{7}$$

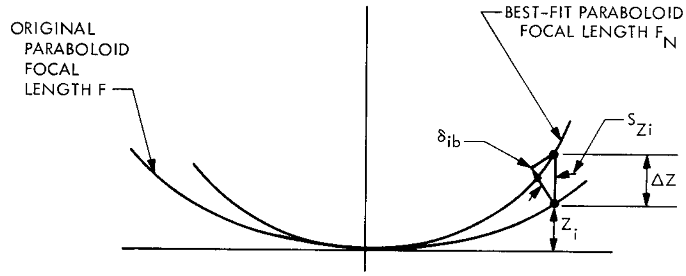


**Fig. 3. Normal error from focal length change**

Equation (7) is equivalent to Eq. (4) and is used in the coding of the program.

The third,

$$\begin{aligned} S_{ic} &= \text{normal error due to rigid body translations of the paraboloid} \\ &= -U_0 S_{zi} - V_0 S_{yi} - W_0 S_{zi} \end{aligned} \tag{8}$$

The fourth,

$$\begin{aligned} S_{id} &= \text{normal error due to rigid body rotations of the best-fit paraboloid} \\ &= (z_i S_{yi} - y_i S_{zi}) + \beta\,(x_i S_{zi} - z_i S_{xi}) \end{aligned} \tag{9}$$

The normal error due to positive (right-hand rule) rotation about the $Y$ axis is graphically defined in Fig. 4.

From Fig. 4, the normal error due to rigid body rotation is

$$-\delta = -\beta x_i S_{zi} + \beta z_i S_{xi}$$

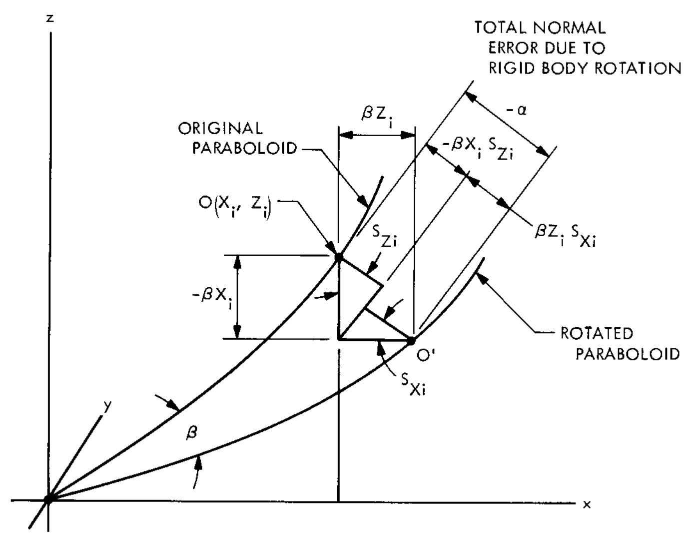


**Fig. 4. Normal error due to rotation about y axis**

which transposes to

$$\delta = \beta\,(x_i S_{zi} - z_i S_{xi})$$

The linear type of calculations for offsets due to rotation is defined in Ref. 2 and, as stated therein, rotations are limited to small angles.

In summing, the ½-path-length change

$$\begin{aligned}\Delta PL_i = {} & [u_i S_{xi} + v_i S_{zi} + w_i S_{zi} - K^1 z_i S_{zi} - U_0 S_{xi} \\ & - V_0 S_{yi} - W_0 S_{zi} + \alpha\,(z_i S_{yi} - y_i S_{zi}) \\ & + \beta\,(x_i S_{zi} - z_i S_{xi})]\, S_{zi} \qquad (10)\end{aligned}$$

Equation (10) is equivalent to the corrected Eq. (8) of Ref. 1, with the exception that $\Delta PL$ now is referenced to ½-path-length change, and as shown in the reference, the best-fit paraboloid is found by minimizing $R$, the sum of the squares of the residuals (i.e., path-length change) where

$$R = \Sigma_i\,(\Delta PL_i)^2\,A_i$$

and where $A_i$ is a weighting factor (usually the area of the surface panel associated with the measured point when a uniform RF illumination density is assumed).

The minimization and the best-fit data of the new paraboloid then result from a solution of a set of six linear normal equations derived from setting the partials of $R$ with respect to the six parameters of motions equal to zero.

A new double-precision subroutine identified in the JPL Fortran V Subroutine Library as DVANAS3—Singular Value Analysis of a Linear Least Squares Problem (Ref. 7) replaces the MATINV subroutine used for the solution of the resulting matrix equation $Ax = b$, where

$$A = \Sigma\,A_i S_{zi}\,\{D\}\,\{D\}^T, \qquad x = C, \qquad b = \Sigma\,A_i S_{zi}^2 y_i\,\{D\}$$

$$D = \left\{\begin{matrix} S_{xi} \\ S_{yi} \\ S_{zi} \\ S_{zi}\,(x_i^2 + y_i^2) \\ (S_{yi} y_i - S_{zi} z_i) \\ (S_{xi} z_i - S_{yi} x_i) \end{matrix}\right\} \qquad C = \left\{\begin{matrix} U_0 \\ V_0 \\ W_0 \\ K \\ \alpha \\ \beta \end{matrix}\right\}$$

$$y_i = S_{xi} m_i + S_{zi} v_i + S_{yi} w_i$$

The DVANA3 subroutine computes and prints a sequence of candidate solutions with their singular values, the sum of the squares of the residuals, and other quantities useful in analyzing a least squares problem.

Preliminary evaluation, based on these quantities, indicates that the matrix is well conditioned for accurate answers of $W_0$ ($z$ offset), $K$ (focal length), $\alpha$ (rotation about $x$ axis) and $\beta$ (rotation about $y$ axis).

It follows that the RMS value is accurately determined. However, the $U_0$ ($x$ offset) and $V_0$ ($y$ offset) answers results from large ratios of singular values, and this requires precise input deflection values ($u_i, v_i, w_i$) in order for $U_0$ and $V_0$ answers to be meaningful. The present interpretation is that the analytically computed deflections provide useful $U_0$ and $V_0$ answers for determining the RF boresight directions and the existing Theodolite-type field measurements produce marginal answers.

Test problems were formulated to determine the linearized formulation error which occurs only for rotations $\alpha$ and $\beta$. For the 210-ft case, where the rotation about the $x$ axis ($\alpha$) was less than 0.003 rad, the rms error was 0.001 in. and $V_0$ displaced 0.004 in. For only translations and focal length changes, the formulation is exact.

**Definition of terms**

| | |
|---|---|
| $u_i, v_i, w_i = x, y, z$ | Components of the distortion vector of point or node $i$ from the original paraboloid |
| $S_{xi}, S_{yi}, S_{zi} = x, y, z$ | Direction cosines of the normal to the original paraboloid |
| $S_{xj}, S_{yj}, S_{zj} = x, y, z$ | Direction cosines of the normal to the best-fit paraboloid. |
| $U_0, V_0, W_0$ | Rigid body translations or vertex offsets of the best-fit paraboloid |
| $S_i =$ | normal component of the distortion vector of point $i$. |
| $\Delta PL_i =$ | ½-path-length change at point $i$. |
| $A_i =$ | RF weighting function of point $i$. |

## References

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